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Effect of Prey Refuge on a Prey- predator model

Anand Soni[1], Jyoti Gupta[2]

[1]*Department of Basic Sciences, Bansal Institute of Science and Technology, Bhopal, India*

Abstract

Prey refuge behavior is the key to sustaining the predator population. The presented model is a prey-predator system assuming logistic prey growth and prey refuge. The author established the existence and the local stability of the equilibriums. The system is stable for no or moderate prey refuge and collapses for its highest value. Biologically, providing a suitable predating environment to predator helps to sustain the predator population.

Keywords: *Stability, Escape rate, Prey, Predator.*

1. Introduction

The evolution of prey and predator interactions is a captivating subject in ecology. While numerous studies focus predominantly on ecological or evolutionary dynamics, real-world prey-predator systems often simultaneously encompass both processes. Experimental research has demonstrated that ecological and evolutionary mechanisms are intricately intertwined, presenting significant challenges in identifying and analyzing their complex interplay within natural systems. Eco-evolution is the reciprocal interaction between ecological dynamics and evolutionary changes, where each influence and shapes the other over time. Theoretically, theoretical models are crucial in facilitating analysis and advancing research [1]. Theoretical models of predator-prey dynamics consistently illustrate that the population sizes of predators and prey exhibit regular and periodic oscillations over time. Typically, predator population density lags behind that of prey. An increase in prey density promotes a corresponding rise in predator population due to the ample prey availability for predation. However, as prey density subsequently declines, the predator population also diminishes. A reduced predator density decreases the predation rate, facilitating the recovery and subsequent increase of the prey population. As a result, both predator and prey species experience continual environmental fluctuations on short temporal scales. From an evolutionary standpoint, these dynamics induce periodic shifts in selective pressures between predator and prey, thereby driving the evolution of adaptive prey defense mechanisms. This study investigates an eco-evolutionary model of prey-predator interaction employing the prey escape rate as an evolutionary parameter to evaluate its impact on ecological dynamics. Warif B. Bassim studied the prey-predator ecosystem from both the prey and predator perspectives and found a hide-and-evade strategy that follows prey, prevents them from predation, and balances the ecosystem [2]. Aposematism is an adequate strategy prey uses to defend themselves and signal potential threats. By employing aposematism, prey can reduce the likelihood of predation, thereby increasing their escape rate [3]. The topic of prey and predator coevolution is highly sophisticated, involving the dynamic adaptation of strategies by both prey and predators in response to their compatibility and prevailing environmental conditions [4]. In this mathematical model, the author investigated the oscillatory behavior of populations using partial differential equations. The model examines the defense traits developed by the prey population, which significantly enhance their chances of survival. These defense traits, favored within the prey population, contribute to increased survivability while maintaining a low energy cost. Prey defend themselves from predation by developing various defense mechanisms, such as behavioral adaptations and the release of chemical defenses. These strategies are often unfavorable for predators and, in some cases, may lead to their death, potentially destabilizing the ecosystem [5].

The proposed model encapsulates a prey-predator model incorporating prey refuge. Section 2 contains the development of the mathematical model. Section 3 presents equilibrium classification. Section 4 explores stability analysis of the model at various point. Section 5 pursue numerical simulation of the proposed model. Finally, section 6 concludes results.

2. Development of the Mathematical Model

The proposed model considers a prey N_p and predator species N_c. The prey population is growing according to logistic rule. We assume that predating is predating on prey with linear function response and conversing the predator

E-mail ID: drjyotigupta@bistbpl.in

population. Further, some preys are refuging predation using their aposematic behavior. Considering all above assumption the proposed model is:

$$\frac{\mathrm{dN_c}}{\mathrm{dt}} = rN_p\left(1-\frac{N_p}{k}\right) - aN_pN_c(1-e) \qquad (1)$$

$$\frac{\mathrm{dN_c}}{\mathrm{dt}} = \mathrm{baN_pN_c(1-e) - mN_c}$$

The parameters and functions are defined in the following table:

No.	Parameters	Definition
1	N_p	Population of preys
2	N_c	Population of predators
3	r	Reproduction rate of prey
4	k	Carrying capacity of prey
5	a	Predation rate (prey captured per predator per unit time)
6	m	Mortality rate of predators
7	b	Conversion efficiency of predators (prey to predator biomass)
8	e	Prey escape rate (evolutionary parameter)

Prey escape rate $e \in [0,1]$, represents the fraction of prey escaping predation using some evolved traits (like speed, camouflage) during the process of predation. $e = 0$ means no prey escape, $e = 1$ means all prey escape.

3. Classification of Equilibrium Points

The Equilibrium points are: (a) Trivial equilibrium $(0,0)$ (b) Axial equilibrium point: $(N_p, 0)$ (c) Coexistence equilibrium point(N_p, N_c): Where, $N_p = \frac{m}{ba(1-e)}$ and $N_c = \frac{r}{a(1-e)}\left(1-\frac{m}{ba(1-e)k}\right)$.

4. Local Stability Analysis

The Jacobian of system (1) is as follows:

$$J = \begin{pmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{pmatrix}. \qquad (2)$$

Where $u_{11} = r\left(1-\frac{N_p}{k}\right) - r\frac{N_p}{k} - aN_c(1-e)$, $u_{12} = \mathrm{aN_p(1-e)}$, $u_{21} = baN_c(1-e), u_{22} = baN_p(1-e) - m$.

4.1. Trivial Equilibrium: At trivial equilibrium (0,0) the Jacobian (2) becomes

$$J = \begin{pmatrix} r & 0 \\ 0 & -m \end{pmatrix}$$

The eigen values are $\lambda_1 = r, \lambda_2 = -m$. Since $\lambda_1 = r > 0$, the trivial equilibrium point is unstable.

4.2 Axial Equilibrium: At axial equilibrium $(N_p, 0)$ the Jacobian (2) becomes

$$J = \begin{pmatrix} r\left[1-\frac{2m}{ba(1-e)k}\right] & -\frac{m}{b} \\ 0 & 0 \end{pmatrix} \qquad (3)$$

The characteristic equation for the Jacobian (3) is:

$$\left(r\left[1-\frac{2N_p}{k}\right] - \lambda\right)(-\lambda) = 0.$$

The eigen values are,

$$\lambda_1 = r\left[1 - \frac{2m}{ba(1-e)k}\right], \qquad \lambda_2 = 0.$$

Since 0<e<1 therefore λ_1is negative under the condition 2m>ba(1-e)k and λ_2=0, indicates that stability analysis is inconclusive and the equilibrium might be neutrally stable or undergo a bifurcation. Hence axial equilibrium is unstable.

4.3 Coexistence Equilibrium: The Jacobian of the system (1) at coexistence equilibrium (N_p, N_c) is

$$J = \begin{pmatrix} \frac{-mr}{ba(1-e)k} & -\frac{m}{b} \\ br\left(1 - \frac{m}{ba(1-e)k}\right) & 0 \end{pmatrix} \tag{4}$$

The characteristic equation is for the Jacobean (4) is:

$$\lambda^2 + \frac{mr}{ba(1-e)k}\lambda + mr\left(1 - \frac{m}{ba(1-e)k}\right) = 0.$$

The eigen values are,

$$\lambda_1 = \frac{1}{2}\left[\frac{-mr}{ba(1-e)k} - \sqrt[2]{\frac{m^2r^2}{b^2a^2(1-e)^2k^2} - 4mr\left(1 - \frac{m}{ba(1-e)k}\right)}\right]$$

$$\lambda_2 = \frac{1}{2}\left[\frac{-mr}{ba(1-e)k} + \sqrt[2]{\frac{m^2r^2}{b^2a^2(1-e)^2k^2} - 4mr\left(1 - \frac{m}{ba(1-e)k}\right)}\right]$$

Since $0 < e < 1$ therefore λ_1 is always negative. And λ_2 is negative under the condition $m > ba(1-e)k$. Hence coexistence equilibrium is stable under condition $m > ba(1-e)k$.

5. Numerical Simulation

The proposed model is a pre-predator system that considers the logistic prey growth rate and prey refuge coefficient. The author also simulates the model (1) with the following set of biologically realistic parameters:

r = 4.5, k=50, a = 0.35, a = 0.05, b = 0.51, m = 0.1, e=0.5 (5)

The solution trajectories of the system (1) are shown at interior state showing local stability Fig. (1). Further, increasing prey escape rate to e =1 the predator population dies and the system reaches to axial state r Fig. 2. The impact of prey refuge on the predator population is shown in Fig. 3. The predator population is highest at prey refuge rate $e = 0.8$.

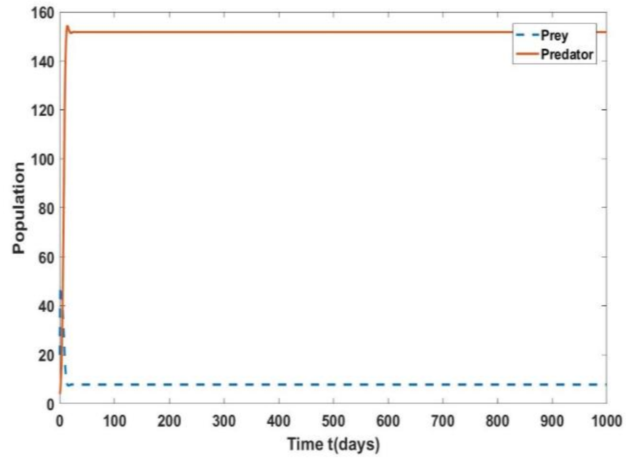

Fig.1: Local Stability of the Coexistence equilibrium for the parameter set (5)

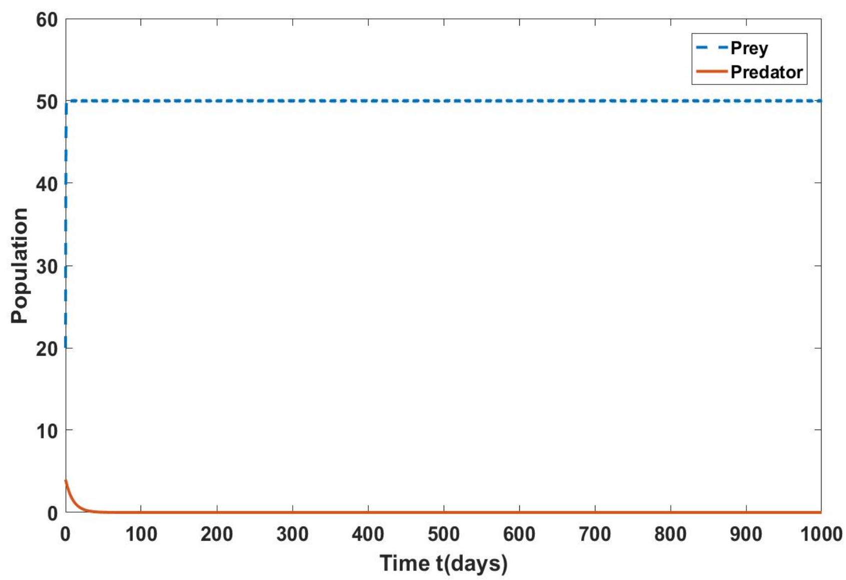

Fig.2 Local Stability of the Axial equilibrium for the parameter set (5) except $e = 1$

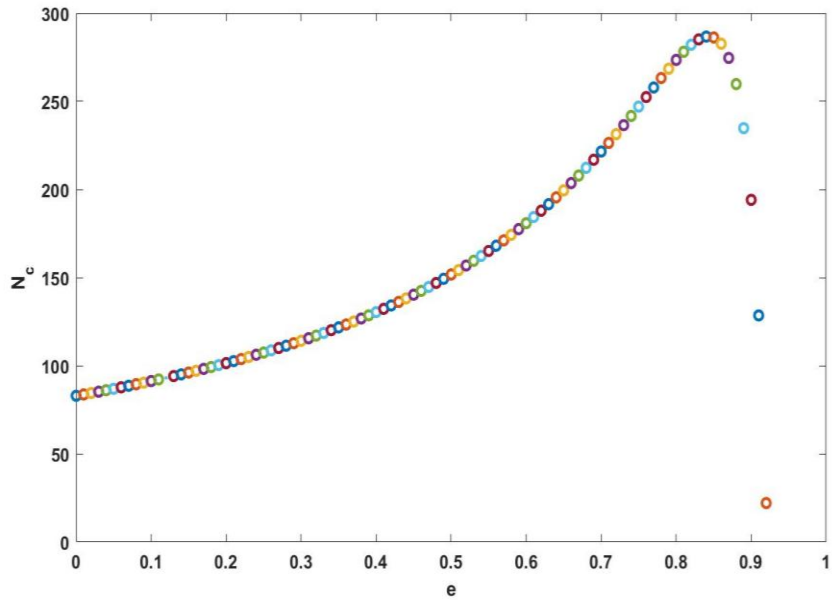

Fig.3 Effect of prey refuge parameter on the praetor population.

6. Conclusions

This section presents the results of the proposed system (1). The author proved analytically the presence of the equilibrium and their local stability. The system (1) is locally stable when there is no and moderate prey refuge. When the prey refuge reaches its highest value, predators are drastically decline and disappear from the system.

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